

Modeling the Conditional Dependence between Stock Market Returns with a Copula-GARCH Approach

Analisi della Dipendenza Condizionata tra i Mercati Azionari mediante un Modello Copula-GARCH

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Riassunto: Recenti ricerche si sono indirizzate verso lo studio delle dinamiche che caratterizzano i legami tra i mercati azionari. Nell'ambito di tali analisi, emerge che la correlazione presentata dai rendimenti dei maggiori indici di mercato azionario non è costante nel tempo e risulta condizionata alle informazioni passate. Il presente lavoro indaga tali dinamiche mediante un approccio Copula-GARCH. I vantaggi dell'adozione di tale approccio risiedono nell'utilizzo di strumenti flessibili di analisi della distribuzione congiunta dei rendimenti (funzioni copula) caratterizzati da un parametro time-varying condizionatamente al passato, con una separata specificazione della forma delle marginali mediante l'applicazione di modelli T-GARCH.

Keywords: Copula function, Correlation, T-GARCH model.

1. Introduction

Stock markets of different size, structures and geographic location can exhibit a high degree of co-movement after a shock in one market, that implies the existence of mechanisms through which domestic shock are transmitted internationally. Actually, it has been demonstrated, see e.g. Engle (2002), Tse and Tsui (2002), that cross market correlation coefficients are conditional on market volatility and during crises, when markets are more volatile, estimates of this correlation tend to increase. This suggests a time-varying conditional dependency. The aim of this paper is to investigate the presence of a conditional dependence between international markets and to study the dynamics of the changes using a conditional approach Copula-GARCH. This framework allows to describe accurately the univariate margins with conditional and time varying parameters and offers, by the copula tool, the advantage to link univariate distributions in a more flexible way rather than adopting the multivariate GARCH analysis. The methodology adopted consists in modeling stock index distributions with a GARCH model, in a univariate context, and in using a copula function to join the margins into a multivariate distribution with a conditional and time varying parameter. For several copula functions, in particular elliptical class of copula, the estimated parameter is the correlation coefficient. For another class of copula family, the archimedean copula, the parameter is a function of Spearman's rho and Kendall's tau. The empirical analysis regards daily data of three stock indexes of market returns:

S&P500, DAX and MIB30 over a period that goes from January 17th, 1995 to May 2nd, 2003. In this framework, we have used the IMF method, estimating in separate steps the T-GARCH and the copula parameters. Firstly, the marginal distributions of each stock index are separately estimated via maximum likelihood through a T-GARCH model with innovations distributed as Student's t . It is well known that the residuals obtained for a GARCH model are generally non-normal and a TGARCH is used because the impact of positive and negative shocks is asymmetric on the conditional volatility (leverage effect). The T-GARCH parameter estimates suggest that the effect of positive returns is larger than negative with a high persistence parameter c . After transforming the standardized residuals into uniform margins, we have estimated for each market pair two bivariate copula functions by MLE: the Clayton, with tail dependence parameter α and the Student's t copula function, with tail dependence when $\rho \neq 1$. In order to choose the copula that presents the better fit with respect to the data, we have compared the log-likelihood functions through the AIC and SIC criteria. Estimating the copula functions, we have assumed two alternative formulations: with fixed parameters and with time-varying parameter, adopting the approach proposed by Tse and Tsui (2000). The dynamic models for the Student's t copula parameter ρ (see Jondeau and Rockinger, 2006) and for the Clayton copula parameter α are given by:

$$\rho_t = (1 - \theta_1 - \theta_2) \rho + \theta_1 \psi_{t-1} + \theta_2 \rho_{t-1} \quad \text{with } 0 \leq \theta_i \leq 1 \text{ and } \sum_i \theta_i \leq 1$$

$$\alpha_t = (1 - \theta_1 - \theta_2) \alpha + \theta_1 \tau_{t-1} + \theta_2 \alpha_{t-1} \quad \text{with } \alpha \in [-1, 0) \cup (0, \infty],$$

where ψ_{t-1} is the Pearson's correlation coefficient and τ_{t-1} is the Kendall's coefficient, at time $t-1$, computed over the last m observations. Figure 1 reports the dynamics of estimated parameters for SPX-DAX ($m=20$).

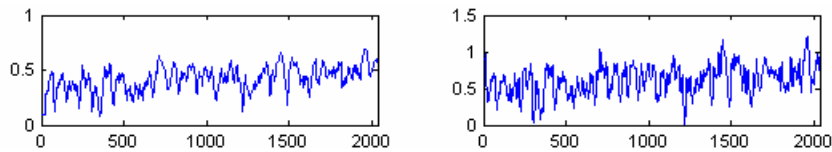


Figure 1: SPX -DAX : dynamic of the Student's t copula parameter ρ (left) and Clayton copula parameter α (right).

References

- Engle, R.F., (2002), Dynamic conditional correlation: a simple class of multivariate generalized autoregressive conditional heteroskedasticity models, *Journal of Business and Economic Statistics*, 20 (3), 339-350.
- Jondeau, E., Rockinger, M., (2006), The Copula-GARCH Model of Conditional Dependencies: an International Stock Market Application, *Journal of International Money and Finance*, 25, 827-853.
- Tse, Y.K., Tsui, A. K.C. (2002) A Multivariate Generalized Autoregressive Conditional Heteroscedasticity model with time-varying correlations, *International Journal of Economics Review*, 39(4), 863-883.